THE SAGE ENGINE

RICHARD KOCH

1. Sage

Sage is an open source alternative to Maple, Mathematica, and Matlab. It can be downloaded from `https://www.sagemath.org`. At the time this report was written, that site directs Macintosh users to `https://github.com/3-manifolds/Sage_macOS/releases` for a Macintosh version which contains Intel code but also runs under emulation on Arm machines. The program provides all of the standard features expected from such a system: arbitrary precision arithmetic, symbolic integration and differentiation, two-dimensional plotting of functions, matrix algebra, and much more.

The Macintosh version of Sage 9.4 has been reconfigured, so it is important to use version 9.4 or later with this engine. When Sage 9.4 was first released, the Macintosh version had serious bugs. These were fixed by October, 2021; be sure to use the fixed version.

To install, double click the dmg file to open it, and drag SageMath-9-4.app to your /Applications folder. Then double-click the install package in the dmg to install a small number of other programs.

Sage is updated regularly, so by the time you read this document there may be a later version available. This document will pinpoint changes you may need to make in our installation for a later version.

2. Installing the Sage Typesetting Engine and SageTeX

To use Sage, drag the sage.engine file in this folder to $\sim$ */Library/TeXShop/Engines*, the folder of active engines for TeXShop. This step need only be done once, when you first install Sage. Unless the structure of Sage changes drastically, this engine should work with later versions of Sage as well.

The Sage program comes with a folder of TeX style files named *sagetex*. These style files allow Sage code to be embedded in a LaTeX document. During typesetting, Sage is called to process the Sage code, and the output is included in the pdf output file. This is an easy way to add graphs of functions to documents, perform symbolic integration, and include intricate numerical calculations.

The one tricky step is to find the sagetex folder in the Sage application and copy it to a location where TeX will find it. Since sagetex depends strongly on the particular version of Sage being used, this step must be repeated each time Sage is updated on your system.

Date: October 3, 2021.

We will copy the sagetex folder to ∼/Library/texmf/tex/latex. Here ∼/Library is the Library folder in your home directory, not the system Library folder, and Apple usually hides the folder. But TeXShop has a command which opens ∼/Library/TeXShop. Create the texmf/tex/latex folders inside ∼/Library if they do not currently exist.

In the folder containing the document you are now reading, there is a folder named "Sage and Latexmk" That folder contains the sagetex folder for SageMath-9-4. If you have version 9-4, drag and drop this sagetex folder to the location ∼/Library/texmf/tex/latex/ while holding down the Option key so you only make a copy of the folder.

If you have a later version of SageMath, the program itself contains the sagetex folder for that version. Follow the steps below to obtain this folder. These instructions list the version number of Sage. Change the two occurrences of 9-4 to the later version number for future versions of Sage.

```
cd /Applications/SageMath-9-4.app/Contents/Frameworks/Sage.framework
cd Versions/9.4/local/share/texmf/tex/latex
```

These commands set the current Terminal directory to the location of sagetex. Issue the command

```
open .
```

to open this location in the Finder; notice the period at the end of this command. The result will be a Finder window containing the sagetex folder. Drag and drop this sagetex folder to the location ∼/Library/texmf/tex/latex/ while holding down the Option key so you only make a copy of the folder.

3. How SageTeX Works

This document ends with sample source code and output illustrating how sagetex works. In the source file, the initial line

```
% !TEX TS-program = sage
```

tells TeXShop to process the file using the sage engine; this engine first calls pdflatex, then calls sage, and finally calls pdflatex again. The remaining lines in the preamble are standard LaTeX commands, except the required line

```
\usepackage{sagetex}
```

In the remaining source, sage commands are entered within lines of the form

```
\sage{....}
```

These lines cause sage to process commands and output LaTeX source fragments, which become part of the LaTeX document.

Notice in particular that sage can plot standard functions. Sage can also compute integrals symbolically; for example, look carefully at the command which processes $\int \frac{x^2+x+1}{(x-1)^3(x^2+x+2)}$.

This command contains standard LaTeX code to display the integral, but then Sage integrates and returns a typeset copy of the result.

4. Another Sample

The "Sage and Latexmk" folder contains a more extensive sample file called "example.tex" by Dan Drake. That sample is set up to use pdflatexmk, but it also works with the standard sage engine by changing the word "pdflatexmk" to "sage" on the top line. Make a copy of this file in a separate folder and typeset to try out Sage.

5. A Debugging Warning

After one update to Sage, the sample document included in this folder stopped working. It turned out that the syntax for one Sage command had changed slightly. Breaking just one sage command caused them all to fail. Consequently, if you intend to use Sage together with TeX and suddenly nothing works, a little clever debugging will be required to determine and fix the bad Sage command.

6. Final Remarks

A Sage tutorial is available at the Sage page `https://www.sagemath.org`. It is definitely recommended. Extensive additional documentation is available at the same web page.

```
% !TEX TS-program = sage

% The following lines are standard LaTeX preamble statements.
\documentclass[11pt, oneside]{amsart}
\usepackage{geometry}
\geometry{letterpaper}
\usepackage[parfill]{parskip}
\usepackage{graphicx}
\usepackage{amssymb}
\usepackage{epstopdf}
\title{Brief Article}
\author{The Author}

% Only one command is required to use Sage within the LaTeX source:
\usepackage{sagetex}

\begin{document}
\maketitle
\section{Introduction}

This is an example of using Sage within a \TeX\ document. We can compute extended values like

      $$32^{31} = \sage{32^31}$$

We can plot functions like $x \sin x$:

 \sageplot[width=4in]{plot(x * sin( 30 * x), -1, 1)}

 We can integrate:

 $$\int {{x^2 + x + 1} \over {(x - 1)^3 (x^2 + x + 2)}}\,dx$$ $$=  \sage{integrate( (x^2 + x + 1) / ((x - 1)^3 * (x^2 + x + 2)), x )}$$

 \newpage
 We can perform matrix calculations:

$$\sage{matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]])^3}$$

$$AB=  \sage{Matrix([[1, 2], [3, 4]])} \sage{Matrix([[5, 6], [6, 8]])} = \sage{Matrix([[1, 2], [3, 4]]) * Matrix([[5, 6], [6, 8]])}
$$

Plots are fun; here is a second one showing $x \ln x$. The ``width'' command in the source is sent to the include graphics command in LaTeX rather than to Sage.

\sageplot[width=5in]{plot(x * ln(x), 0, 2)}

Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term $e \pi$ below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of $e$ and $\pi$ is $\sage{pi * e} = \sage{N(pi * e)}.$

\end{document}
```

BRIEF ARTICLE

THE AUTHOR

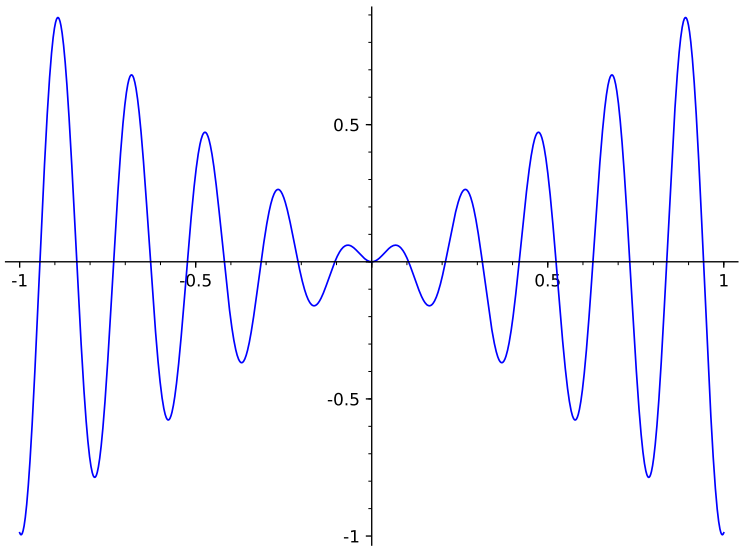
1. Introduction

This is an example of using Sage within a TEX document. We can compute extended values like

$$32^{31} = 45671926166590716193865151022383844364247891968$$

$$1324^{9} = 1250235613859132234566755942 4$$

We can plot functions like $x \sin x$:



We can integrate:

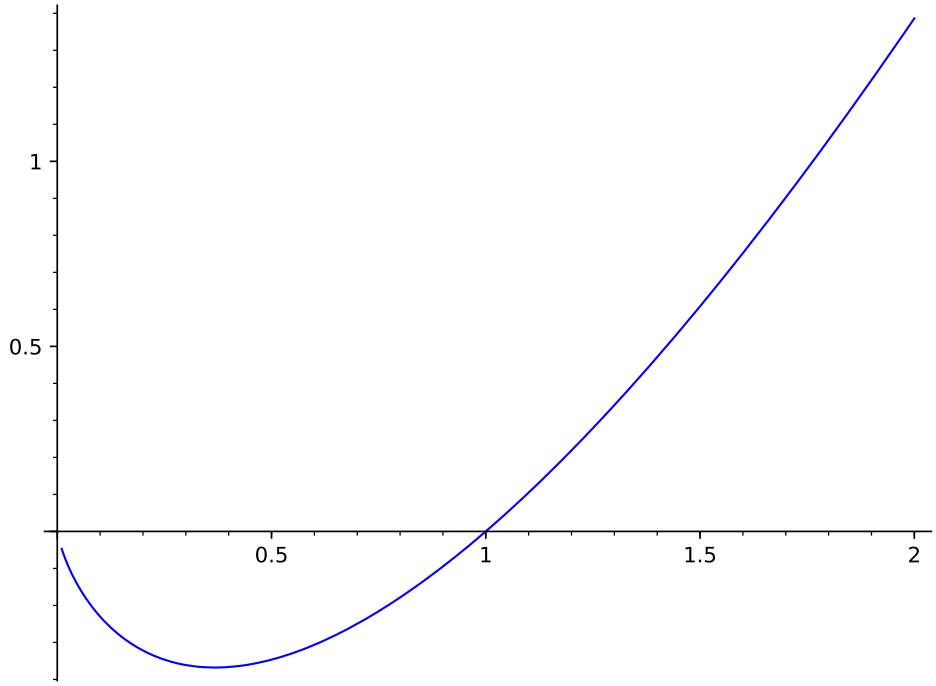
$$\int \frac{x^2+x+1}{(x-1)^3(x^2+x+2)}\,dx$$
$$= -\frac{9}{448}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x+1)\right) - \frac{3\,(x+1)}{16\,(x^2-2\,x+1)} + \frac{5}{128}\log\left(x^2+x+2\right) - \frac{5}{64}\log\left(x-1\right)$$

We can perform matrix calculations:

$$\begin{pmatrix} 468 & 576 & 684 \\ 1062 & 1305 & 1548 \\ 1656 & 2034 & 2412 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 17 & 22 \\ 39 & 50 \end{pmatrix}$$

Plots are fun; here is a second one showing $x \ln x$. The "width" command in the source is sent to the include graphics command in LaTeX rather than to Sage.



Sage understands mathematical constants and writes them symbolically unless it is told to produce a numerical approximation. The term $e\pi$ below is not in the LaTeX source; instead it is the result of a Sage calculation, as is the numerical value on the other side of the equal sign.

The product of e and π is $\pi e = 8.53973422267357$.